一、数据及处理

1. 共振频率的测量结果：$f_0 = 39.080\,kHz$，$\sigma_{f_0} = \frac{0.001}{\sqrt{3}}\,kHz$

2. 逐差法　温度 $\theta = 25.5°C$　压强 $P = 1.01666\times10^5\,Pa$

输入电压幅值 4.000 V

缩小间距，测出正弦波振幅极大值之位置及对应的峰峰值电压

i	1	2	3	4	5	6	7	8	9	10
x_i/mm	60.640	58.142	50.200	46.072	41.598	36.850	32.089	27.526	23.577	19.000
V_i/V	0.580	0.652	0.708	0.788	0.952	1.08	1.34	1.52	1.79	2.26

增大换能器间距

i'	1	2	3	4	5	6	7	8	9	10
x_i'/mm	19.017	23.361	27.565	32.022	36.710	41.165	46.047	50.218	56.138	60.202
V_i'/V	2.26	1.90	1.66	1.46	1.13	0.992	0.864	0.824	0.696	0.608
$\bar{x}_i$/mm	19.0085	23.4690	27.5455	32.0555	36.7800	41.3815	46.0595	50.2090	56.1400	60.4210
$\bar{x}_{i+5}-\bar{x}_i$/mm	22.3730	22.5905	22.6635	24.0845	23.6410					

$\langle 5\times\frac{\lambda}{2}\rangle = 23.0705\,mm$　$\sigma_A = 0.34\,mm$　$\sigma_B = \frac{0.004}{\sqrt{3}}\,mm$

$\Rightarrow \lambda = 9.23\,mm$　$\sigma_\lambda = \frac{2}{5}\sqrt{\sigma_A^2+\sigma_B^2} = 0.14\,mm$

$v = \lambda\cdot f_0 = 360.71\,m/s$　$\sigma_v = v\sqrt{(\frac{\sigma_\lambda}{\lambda})^2+(\frac{\sigma_f}{f_0})^2} = 6\,m/s$

$\therefore v = (361\pm6)\,m/s$

3. 相位法　温度 $\theta = 25.5°C$　压强 $P = 1.01666\times10^5\,Pa$

增大间距

i	1	2	3	4	5	6	7	8	9	10
x_i/mm	17.060	26.558	35.664	44.660	53.564	62.515	71.469	80.452	89.229	98.380

缩小间距

i'	1	2	3	4	5	6	7	8	9	10
x_i'/mm	98.329	89.201	80.560	71.545	62.515	53.569	44.650	35.646	26.550	17.075

i	1	2	3	4	5	6	7	8	9	10
$\bar{x}_i$/mm	17.0675	26.5540	35.6550	44.6550	53.5665	62.5150	71.5070	80.5060	89.2150	98.3595
$\bar{x}_{i+5}-\bar{x}_i$/mm	45.4475	44.9530	44.8510	44.8600	44.7875					

$\langle \bar{x}_{i+5}-\bar{x}_i \rangle = 44.9198\ mm \quad \sigma_A = 0.15mm \quad \sigma_B = \frac{0.004mm}{\sqrt{3}}$

$= \langle 5\lambda \rangle$

$\Rightarrow \lambda = 8.98\ mm \quad \sigma_\lambda = \frac{1}{5}\sqrt{\sigma_A^2+\sigma_B^2} = 0.03\ mm$

$v = \lambda f_0 = 350.94 m/s \quad \sigma_v = v\sqrt{(\frac{\sigma_\lambda}{\lambda})^2+(\frac{\sigma_f}{f_i})^2} = 1.2 m/s$

$\therefore v = (350.9 \pm 1.2)m/s$

上面为逐差法处理所得，下面将使用最小二乘法进行处理

$x_i = \lambda \cdot i + x_0 \Rightarrow \lambda = 8.9935\ mm, \quad x_0 = 8.494$

$r = 0.999976$

$\frac{\sigma_{\lambda 1}}{\lambda} = \sqrt{\frac{1/r^2-1}{10-2}} = 0.0025$

$\sigma_{\lambda 1} = 0.023\ mm$

$\sigma_{\lambda 2} = \frac{\sigma_{xi}}{\sqrt{\sum_{i=1}^{10}(i-5.5)^2}} = \frac{0.004mm/\sqrt{3}}{\sqrt{\sum_{i=1}^{10}(i-5.5)^2}} = 0.00025mm$

$\sigma_\lambda = \sqrt{\sigma_{\lambda 1}^2+\sigma_{\lambda 2}^2} = 0.023mm$

$\Rightarrow \lambda = (8.994 \pm 0.023)mm$

$v = \lambda f = 351.48 m/s$

$\sigma_v = v\cdot\sqrt{(\frac{\sigma_\lambda}{\lambda})^2+(\frac{\sigma_f}{f})^2} = 0.9 m/s$

$\therefore v = (351.5 \pm 0.9)m/s$

4. 气体参量法

温度 $\theta = 25.5°C \quad \sigma_\theta = \frac{1°C}{\sqrt{3}}$

饱和蒸气压 P_s：$25°C: 3167.6Pa \quad 26°C: 3361.3Pa$

$P_s = (\frac{3361.3-3167.6}{26-25}(\theta-25)+3167.6) Pa = 3264.45Pa$

$\sigma_{P_s} = \frac{3361.3-3167.6}{26-25}\sigma_\theta = 1.12\times10^2 Pa$

$P_s = (3.264 \pm 0.112)\times10^3 Pa$

相对湿度：干湿球湿度器：$H_1 = 54\%$，家用湿度器：$H_2 = 58\%$

$H = 56\% \pm 4\%$

压强：保护管上湿度计读数 $t = 25°C$，$\sigma_t = \frac{1°C}{\sqrt{3}}$

水银柱高度得压强：$P_1 = 766.25mmHg, \quad \sigma_{P_1} = \frac{0.05mmHg}{\sqrt{3}}$

温度修正：$P_2 = P_1 - (0.000182 - 1.00\times10^{-5})P_1 t$

重力加速度修正：北京地区重力加速度 $g = 9.8015 m/s^2$

$P = \frac{g}{g_0}P_1[1-(0.000182-1.00\times10^{-5})t] = 762.55mmHg = 101665.6Pa$

$\sigma_P = P\cdot\left[(\frac{\sigma_{P_1}}{P_1})^2+(\frac{(0.000182-1.00\times10^{-5})\sigma_t}{1-(0.000182-1.00\times10^{-5})t})^2\right]^{1/2} = 10.8Pa$

$\therefore P = (1.01666 \pm 0.00011)\times10^5 Pa$

声速：$v(m/s) = 331.45\sqrt{(1+\frac{\theta}{T_0})(1+\frac{0.3192HP_s}{P})} = 347.57 m/s$

为方便计算不确定度，记 $A = 1+\frac{\theta}{T_0}$；$B = 1+\frac{0.3192HP_s}{P}$

有 $\sigma_v = v\sqrt{(\frac{\sigma_A}{2A})^2 + (\frac{\sigma_B}{2B})^2}$

$\sigma_A = \frac{\sigma_\theta}{T_0}$

$\sigma_B = \frac{0.3192HPs}{P}\sqrt{(\frac{\sigma_H}{H})^2 + (\frac{\sigma_{Ps}}{Ps})^2 + (\frac{\sigma_P}{P})^2}$

$\Rightarrow (\frac{\sigma_A}{2A})^2 = 9.3\times10^{-7}$ $(\frac{\sigma_B}{2B})^2 = 5.1\times10^{-8}$

$\Rightarrow \sigma_v = 0.4\ m/s$

$\therefore v = (347.6 \pm 0.4)\ m/s$

5. 水中声速，相位法 $\theta_{H_2O} = 23.5°C$ 水温

共振频率：$f_{H_2O} = 775kHz$ $\sigma_f = \frac{1kHz}{\sqrt{3}}$

增大间距

i	1	2	3	4	5	6	7	8	9	10
x_i/mm	6.605	8.610	10.620	12.570	14.519	16.452	18.440	20.360	22.333	24.285

减小间距

i'	1	2	3	4	5	6	7	8	9	10
x_i'/mm	24.199	22.365	20.395	18.478	16.500	14.576	12.603	10.675	8.695	6.669
$\bar{x_i}$/mm	6.6370	8.6525	10.6475	12.5865	14.5475	16.4760	18.4590	20.3775	22.3490	24.242
$\bar{x_{i+5}} - \bar{x_i}$/mm	9.8390	9.8065	9.7300	9.7635	9.6945					

$\langle \bar{x_{i+5}} - \bar{x_i} \rangle = \langle 5\lambda \rangle = 9.7665\ mm$ $\sigma_A = 0.026mm$, $\sigma_B = \frac{0.004}{\sqrt{3}}mm$

$\Rightarrow \lambda = 1.9533\ mm$ $\sigma_\lambda = \frac{1}{5}\sqrt{\sigma_A^2 + \sigma_B^2} = 0.006\ mm$

$\lambda = (1.953 \pm 0.006)\ mm$

$v = \lambda f = 1513.575\ m/s$

$\sigma_v = v\cdot\sqrt{(\frac{\sigma_\lambda}{\lambda})^2 + (\frac{\sigma_f}{f})^2} = 5\ m/s$

$\therefore v = (1514 \pm 5)\ m/s$ 为23.5°C下水中声速

二、分析与讨论

1、振幅极大值处峰峰值随距离关系

i	1	2	3	4	5	6	7	8	9	10
$\bar{x}_i$/mm	19.0085	23.4690	27.5455	32.0555	36.7800	41.3815	46.0595	50.2090	56.1400	60.4210
$\bar{V}_i$/V	2.260	1.845	1.590	1.400	1.105	0.9720	0.8260	0.7660	0.6740	0.5940
$\ln\bar{V}_i$	0.8154	0.6125	0.4637	0.3365	0.09984	-0.02840	-0.1912	-0.2666	-0.3945	-0.5209

最小二乘拟合 $\ln\bar{V}_i = ax_i + b$

$\Rightarrow a = -30.798\ \text{mm}^{-1}$

$b = 42.16$

$r = -0.993$.

$\therefore V_i = V_0 \cdot e^{-\mu x}$, $\mu = 30.798\ \text{mm}^{-1}$