

测定金属的杨氏模量

一、实验数据与处理

测量金属丝受外力拉伸后的伸展变化数据表

i	m_i/g	r_i/mm	r'_i/mm	\bar{r}_i/mm
0	0.00	2.85	2.85	2.850
1	200.00	2.96	2.94	2.950
2	400.00	3.07	3.05	3.060
3	600.00	3.18	3.16	3.170
4	800.00	3.29	3.29	3.290
5	1000.00	3.40	3.39	3.395
6	1200.00	3.51	3.50	3.505
7	1400.00	3.62	3.61	3.615
8	1600.00	3.74	3.73	3.735
9	1800.00	3.85	3.85	3.850

换按上表砝码顺序测量砝码质量

i	1	2	3	4	5	6	7	8	9
m/g	200.00	199.75	199.90	199.92	199.32	199.79	199.62	199.85	200.69

$$\bar{m} = \frac{1}{9} \sum_{i=1}^9 m_i = 199.87g \quad \sigma_{Am} = \sqrt{\frac{1}{9 \times 8} \sum_{i=1}^9 (m_i - \bar{m})^2} = 0.12g$$

$$\sigma_{Bm} = \frac{0.01g}{\sqrt{3}} \quad \sigma_m = \sqrt{\sigma_{Am}^2 + \sigma_{Bm}^2} = 0.12g \Rightarrow m = (200.00 \pm 0.12)g$$

$$\text{金属丝有效长度: } L = 103.70cm - 25.00cm = 78.70cm$$

$$\sigma_L = \frac{0.15cm}{\sqrt{3}} = 0.09cm \Rightarrow L = (78.70 \pm 0.09)cm$$

金属丝直径 d .

i	1	2	3	4	5	6	7	8	9	10
d'/mm	0.320	0.319	0.317	0.319	0.319	0.318	0.320	0.320	0.320	0.319

$$\bar{d}' = 0.319 \text{ mm} \quad \sigma_{Ad} = \sqrt{\frac{1}{10 \times 9} \times \sum_{i=1}^{10} (d_i' - \bar{d}')^2} = 0.0003 \text{ mm}$$

$$\sigma_{Bd} = \frac{0.004 \text{ mm}}{\sqrt{3}} \quad \sigma_d = \sqrt{\sigma_{Ad}^2 + \sigma_{Bd}^2} = 0.0023 \text{ mm}$$

零点误差: $d_0 = -0.001 \text{ mm}$

$$\Rightarrow d = (0.320 \pm 0.0023) \text{ mm}$$

二. 逐差法 接上页伸长量表

i	0	1	2	3	4
$(r_{i+5} - r_i) / \text{mm}$	0.545	0.555	0.555	0.565	0.560

$$\langle r_{i+5} - r_i \rangle = 0.556 \text{ mm} \quad \sigma_A = \sqrt{\frac{1}{3 \times 4} \sum [\langle r_{i+5} - r_i \rangle - (r_{i+5} - r_i)]^2} = 0.003 \text{ mm}$$

$$\sigma_B = \frac{0.005 \text{ mm}}{\sqrt{3}} \quad \sigma = \sqrt{\sigma_A^2 + \sigma_B^2} = 0.0044 \text{ mm}$$

$$SL = \frac{1}{5} \langle r_{i+5} - r_i \rangle = (0.1112 \pm 0.0009) \text{ mm}$$

$$E = \frac{4mgL}{\pi d^2 SL} = 1.725 \times 10^{11} \text{ Pa}$$

$$\frac{\sigma_E}{E} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{2\sigma_d}{d}\right)^2 + \left(\frac{\sigma_{SL}}{SL}\right)^2}$$

$$\Rightarrow \sigma_E = 0.029 \times 10^{11} \text{ Pa}$$

$$\therefore E = 1.75 \pm 0.03 (1.725 \pm 0.029) \times 10^{11} \text{ Pa}$$

三. 最小二乘法

$$\bar{r}_i = \frac{4mgL}{\pi d^2 E} i + \bar{r}_0, \quad i=0, 1, \dots, 9$$

$$= ki + b, \quad k = \frac{4mgL}{\pi d^2 E}, \quad E = \frac{4mgL}{\pi d^2 k}$$

$$\Rightarrow k = 1.11394 \times 10^{-4} \text{ m}$$

$$b = 2.8407 \times 10^{-3} \text{ m}$$

$$r = 0.99987$$

$$\sigma_{k1} = k \sqrt{\frac{1/r^2 - 1}{10 - 2}} = 0.0062 \times 10^{-4} \text{ m}$$

$$\sigma_{k2} = \frac{\sigma_r}{\sqrt{\sum_{i=0}^9 (i - 4.5)^2}} = \frac{0.005 \text{ mm} / \sqrt{3}}{\sqrt{\sum_{i=0}^9 (i - 4.5)^2}} = 0.0010 \times 10^{-4} \text{ m}$$

$$\therefore \sigma_k = \sqrt{\sigma_{k_1}^2 + \sigma_{k_2}^2} = 0.007 \times 10^{-4} \text{ m}$$

$$\therefore k = (1.114 \pm 0.007) \times 10^{-4} \text{ m}$$

$$E = \frac{4mgL}{\pi d^2 k} = 1.7219 \times 10^{11} \text{ Pa}$$

$$\frac{\sigma_E}{E} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{2\sigma_d}{d}\right)^2 + \left(\frac{\sigma_k}{k}\right)^2}$$

$$\sigma_E = 0.027 \times 10^{11} \text{ Pa}$$

$$\therefore E = (1.72 \pm 0.02) \times 10^{11} \text{ Pa}$$

四. 误差来源

我们可以看到由最小二乘法所得到的结果比逐差法得到的结果的不确定度略小，说明最小二乘法所得结果更加精确。

在逐差法中， $\frac{\sigma_m}{m} = \frac{0.12 \text{ g}}{200 \text{ g}} = 0.0006$

$$\frac{\sigma_L}{L} = \frac{0.09 \text{ cm}}{78.70 \text{ cm}} = 0.0011$$

$$\frac{2\sigma_d}{d} = 2 \frac{0.0023 \text{ mm}}{0.32 \text{ mm}} = 0.014$$

$$\frac{\sigma_{\delta L}}{\delta L} = \frac{0.0009 \text{ mm}}{0.1112 \text{ mm}} = 0.008$$

可见主要误差是由钢丝直径测量带来的，直径测量带来的

在最小二乘法中， $\frac{\sigma_k}{k} = 0.006$

此时主要误差仍是直径，而伸长量误差略有降低。

为使实验更精准，我们可以选择更精确的方式测量钢丝直径。

钢丝直径可用细丝衍射测量，或能得到更为精准之杨氏模量。



预: 大砝码 $\sim 200g$ 小 $\sim 100g$ $g = 9.801 m/s^2$

$$E = \frac{4\pi g L}{\pi d^2 SL}$$

i	m_i / g	r_i / mm	r_i' / mm	\bar{r} / cm
0	100.0	2.81	2.80	2.805
1	300.0	2.90	2.90	2.90
2	500.0	3.00	3.00	3.00
3	700.0	3.11	3.13	3.12
4	900.0	3.22	3.23	3.225
5	1100.0	3.34	3.34	3.34
6	1300.0	3.45	3.45	3.45
7	1500.0	3.55	3.55	3.55
8	1700.0	3.66	3.65	3.655
9	1900.0	3.76	3.76	3.76

$$\text{金属丝长 } L = 103.7 cm - 24.7 cm = 79.0 cm$$

$$\text{零位 } d_0 = -0.002 mm$$

$$\sigma_B = 0.004 mm$$

i	1	2	3	4	5	6	7	8	9	10	\bar{d}	$\sigma_{d'}$
d' / mm	0.320	0.321	0.320	0.318	0.318	0.319	0.320	0.320	0.318	0.318	0.319	0.0004

$$\bar{d} = (0.321 \pm 0.004) mm$$

$$SL = \frac{3.76 + 3.655 + 3.55 + 3.45 + 3.34 + 3.225 + 3.12 + 3.00 + 2.90 + 2.805}{10}$$

$$= 0.108 mm$$

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逐差

	i	0	1	2	3	4
$(r_{i+5} - r_i) / \text{mm}$		0.545	0.555	0.555	0.565	0.56
						0.560

$$\langle r_{i+5} - r_i \rangle = 0.556 \text{ mm}$$

$$\sigma_A = 0.004 \text{ mm}$$

$$\sigma_B = 0.005 \text{ mm}$$

$$\sigma = 0.007 \text{ mm}$$

$$r_{i+5} - r_i = (0.556 \pm 0.007) \text{ mm}$$

$$\Delta L = \frac{1}{5} (r_{i+5} - r_i) = (0.1112 \pm 0.0014) \text{ mm}$$

$$E = \frac{4mgL}{\pi d^2 \Delta L} = 1.725 \times 10^{11} \text{ Pa}$$

$$\sigma_E = E \cdot \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_{\Delta L}}{\Delta L}\right)^2 + \left(\frac{\sigma_d}{d}\right)^2}$$

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最小二乘:

m/g	0.50	200.50	399.75	599.65	799.57	998.89
\bar{r}_i / mm	2.850	2.950	3.060	3.170	3.290	3.395

m/g	1198.68	1398.30	1598.15	1798.84
\bar{r}_i / mm	3.505	3.615	3.735	3.850

$$\Rightarrow k = 5.575 \times 10^{-4} \text{ m/kg} = \frac{4Lg}{\pi d^2 E}$$

$$\Rightarrow E = \frac{4Lg}{\pi d^2 k} = 1.7203 \times 10^{11} \text{ Pa}$$

$$E = \frac{4mgL}{\pi d^3 SL}, \quad g = 9.801 \text{ m/s}^2$$

$$L = 78.50$$

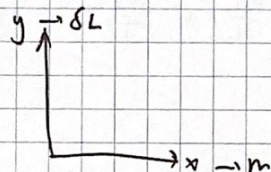
$$e_L = 0.15 \text{ cm} \quad \left. \begin{array}{l} \text{允差 } \pm 0.15 \text{ cm} \\ \text{分度 / 分辨率} \end{array} \right\}$$

$$L \pm e_L = (78.50 \pm 0.15) \text{ cm}$$

$$L \pm \sigma_L = (78.50 \pm 0.09) \text{ cm}$$

$$\sigma_d = \sqrt{\frac{\sum (\bar{d} - d_i)^2}{n(n-1)}}$$

$\frac{12}{4} + 2 = 5$ 误差大 0.0176 12 轴



$$r_i = \frac{4gL}{\pi d^3 E} m_i + b = \frac{4gLm}{\pi d^3 E} \cdot i + b$$

$$\frac{\sigma_{KA}}{K} = \sqrt{\frac{1/r^2 - 1}{n-2}}$$

砝码个数
↓

$$\sigma_{KB} = \frac{\sigma_b}{\sqrt{\sum (x_i - \bar{x})^2}}$$

作图 画成 origin