RLC 电路的谐振现象

2300011403 杨轶

November 2023

1 测量谐振频率与第一种 Q 值

通过李萨如图形，可得出

$$f_0 = 2.2495\,\mathrm{kHz}, \Delta f_0 = 0.0001\,\mathrm{kHz}$$

实验器材参数：标准电阻 $R = 100.00\Omega, \Delta R = 0.01\Omega$；电容箱 $C = 0.05\,\mu\mathrm{F}$，允差约 0.65%；电感 $L = 0.1\,\mathrm{H}$，允差约 0.1%，内阻 $R_L = 18.033\Omega$。

$$Q_1 = \frac{2\pi f_0 L}{R + R_L} = 11.975 \tag{1}$$

$$\sigma_{f_0} = \frac{\Delta_{f_0}}{\sqrt{3}}, \sigma_L = \frac{0.1\% L}{\sqrt{3}}, \sigma_R = \frac{\Delta R}{\sqrt{3}}$$

$$\sigma_{Q_1} = Q_1 \sqrt{(\frac{\sigma_{f_0}}{f_0})^2 + (\frac{\sigma_L}{L})^2 + (\frac{\sigma_R}{R + R_L})^2} = 0.007 \tag{2}$$

$$Q_1 = 11.975 \pm 0.007 \tag{3}$$

2 第二种 Q 值

$$U_{in} = 1.0234\,\mathrm{V}$$

$$U_L = 10.745\,\mathrm{V}$$

$$U_C = 10.715\,\mathrm{V}$$

$$Q_2 = \frac{U_C}{U_{in}} = 10.470 \tag{4}$$

数字多用表交流档允差约为 0.2% + 10 个字

$$\Delta U_{in} = 0.2\% U_{in} + 0.0010 = 0.003, \sigma_{U_{in}} = \frac{\Delta U_{in}}{\sqrt{3}}$$

$$\Delta U_C = 0.2\%U_C + 0.010 = 0.04, \sigma_{U_C} = \frac{\Delta U_C}{\sqrt{3}}$$

$$\sigma_{Q_2} = Q_2\sqrt{(\frac{\sigma_{U_C}}{U_C})^2 + (\frac{\sigma_{U_{in}}}{U_{in}})^2} = 0.05 \tag{5}$$

$$Q_2 = 10.47 \pm 0.05 \tag{6}$$

经对比发现，Q_1 与 Q_2 相差远大于其误差，将通过后面 Q_3 的计算来判断是否合理。

3 相频特性

相差 $\phi = f_i\Delta t$

i	1	2	3	4	5	6	7	8
f_i/kHz	1.000	1.200	1.600	1.800	2.000	2.100	2.200	2.300
Δt/μs	-0.246ms	-0.202ms	-144.5	-120.0	-95.0	-73.0	-32.0	30.5
ϕ/°	-88.56	-87.26	-83.23	-77.76	-68.40	-55.19	-25.34	25.254
i	9	10	11	12	13			
f_i/kHz	2.500	2.700	2.900	3.200	3.600			
Δt/μs	71.0	75.0	74.5	72.0	65.5			
ϕ/°	63.90	72.90	77.79	82.94	84.89			

表 1: 电路相频特性数据表

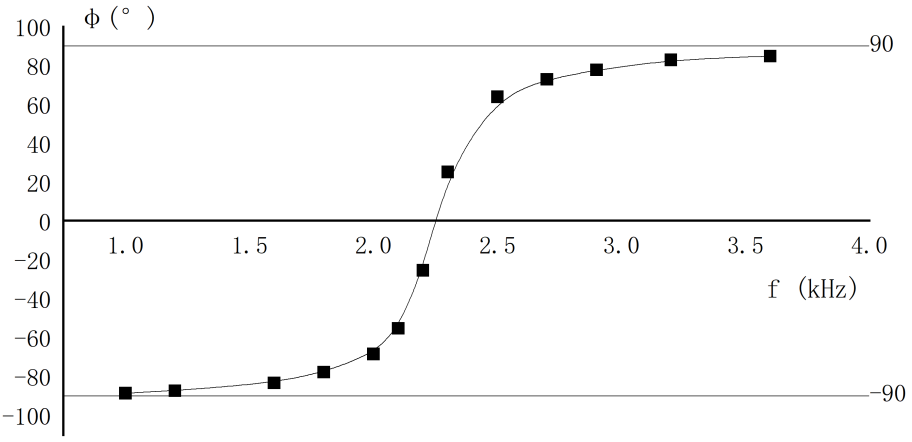

图 1: 相频曲线

4 幅频特性

固定输入电压 $U_{in} = 1V$
电流 $I = \frac{U_R}{R}$

i	1	2	3	4	5	6	7	8
f_i/kHz	1.000	1.200	1.500	1.800	1.900	2.000	2.100	2.150
U_R/mV	39.21	52.68	84.43	153.84	200.52	279.21	0.4257V	0.5442V
I/mA	0.3921	0.5268	0.8443	1.5384	2.0052	2.7921	4.257	5.442
i	9	10	11	12	13	14	15	16
f_i/kHz	2.200	2.220	2.240	2.250	2.260	2.280	2.300	2.350
U_R/V	0.6837	0.7283	0.7534	0.7564	0.7524	0.7264	0.6826	0.5518
I/mA	6.837	7.283	7.534	7.564	7.524	7.264	6.826	5.518
i	17	18	19	20	21	22	23	
f_i/mV	2.400	2.500	2.600	2.800	3.000	3.200	3.600	
U_R/mV	0.4420V	305.38	231.35	156.58	119.43	97.31	72.04	
I/mA	4.420	3.0538	2.3135	1.5658	1.1943	0.9731	0.7204	

表 2: 电路幅频特性数据表

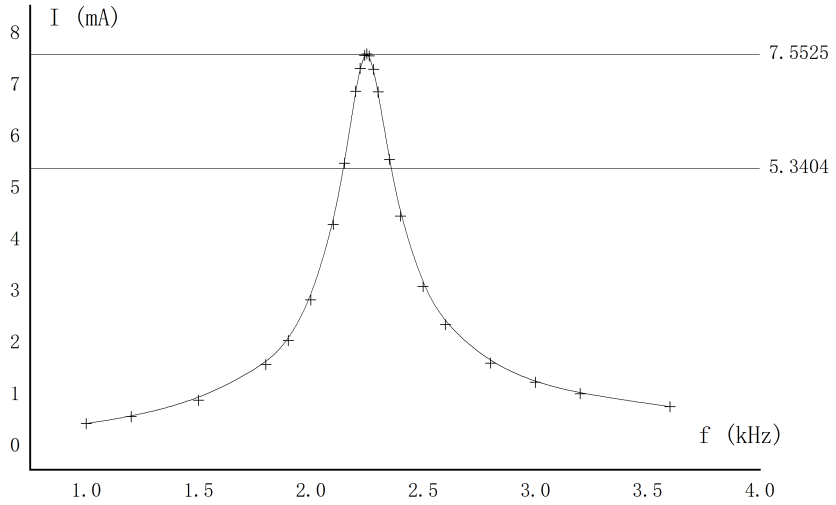

图 2: 幅频曲线

峰值坐标：$f_0 = 2.2494\,\text{kHz}, I_m = 7.5525mA$

$$\frac{I_m}{\sqrt{2}} = 5.3403\,\text{kHz}$$

f_1 与 f_2 对应电流为 $I_m/\sqrt{2}$

$$f_1 = 2.1446\,\mathrm{kHz}, f_2 = 2.3590\,\mathrm{kHz}$$

$$\Delta f = f_2 - f_1 = 0.2144\,\mathrm{kHz}; \tag{7}$$

$$Q_3 = \frac{f_0}{\Delta f} = 10.492 \tag{8}$$

5 关于 Q 值的讨论

我们发现 Q_2 与 Q_3 相差并不大，但 Q_1 与 Q_2, Q_3 有明显的差异。我们认为是由于电路中存在由交流造成的阻抗所导致的。我们可以算出在谐振条件下回路总电阻 R'。

$$\frac{R'}{R} = \frac{U_{in}}{U_R} \tag{9}$$

$$U_{in} = 1.0232\,\mathrm{V}, U_R = 0.7673\,\mathrm{V}$$

$$R' = \frac{U_{in}}{U_R} R = 133.4\Omega \tag{10}$$

姓名 杨敏 学号 23000114013 星期 第 组 页码 1 / 1

1. 谐振频率 f_0 与 Q_1　　$R = 100.00 \pm 0.01\ \Omega$　$L = 0.1\ H$　$R_L = 18.033\Omega$　$C = 0.05\ \mu F$

李萍如图　$f_0 = 2.249\ kHz$　$\Delta f_0 = 0.0001\ kHz$　$\sigma_{f_0} = \frac{\Delta f_0}{\sqrt{3}}$

$Q_1 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = 14.134$　　$\Delta L = 0.1\% \cdot L$　$\sigma_L = \frac{\Delta L}{\sqrt{3}}$

$\sigma_R = \frac{\Delta R}{\sqrt{3}}$

$\sigma_{Q_1} = Q_1 \cdot \sqrt{(\frac{\sigma_{f_0}}{f_0})^2 + (\frac{\sigma_L}{L})^2 + (\frac{\sigma_R}{R})^2} = 0.009$

$\Delta C = 0.65\% \cdot C$

$\sigma_C = \frac{\Delta C}{\sqrt{3}}$

~~2. $Q_2 = \frac{U_L}{U_{in}} = \frac{U_C}{U_{in}}$~~

$Q_1 = \frac{\omega_0 L}{R + R_L} = 11.975$

$Q_1 = \frac{1}{R\omega_0 C} = \frac{1}{2\pi f_0 RC} = 14.150$

$\sigma_{Q_1} = Q_1 \cdot \sqrt{(\frac{\sigma_R}{R})^2 + (\frac{\sigma_{f_0}}{f_0})^2 + (\frac{\sigma_C}{C})^2} = 0.06$

2. $Q_2 = \frac{U_L}{U_{in}} = \frac{U_C}{U_{in}}$

$U_{in} = 1.0281\ V$		$1.0234\ V$
$U_L = 10.625\ V$		$10.745\ V$
$U_C = 10.598\ V$ ✓		~~$10.691\ V$~~ $10.715\ V$

$Q_2 = \frac{U_L}{U_{in}} = 10.334$

$Q_2 = \frac{U_C}{U_{in}} = 10.308$ ✓　　10.470

3. 相频特性

i	1	2	3	4	5	6	7
f_i/kHz	~~1.000~~ 1.000	1.200	1.400	1.600	1.800	2.000	2.100
Δt/ms	−0.246	−0.202	−0.168	~~−144.5μs~~ −144.5μs	−120.0μs	−95.0μs	−73.0μs
~~T/ms~~	~~1.000~~						

i	8	9	10	11	12	13	14
f_i/kHz	2.200	2.300	2.500	2.700	2.900	3.200	3.600
Δt/μs	−32.0	30.5	71.0	75.0	74.5	72.0	65.5

图 3: 课上报告

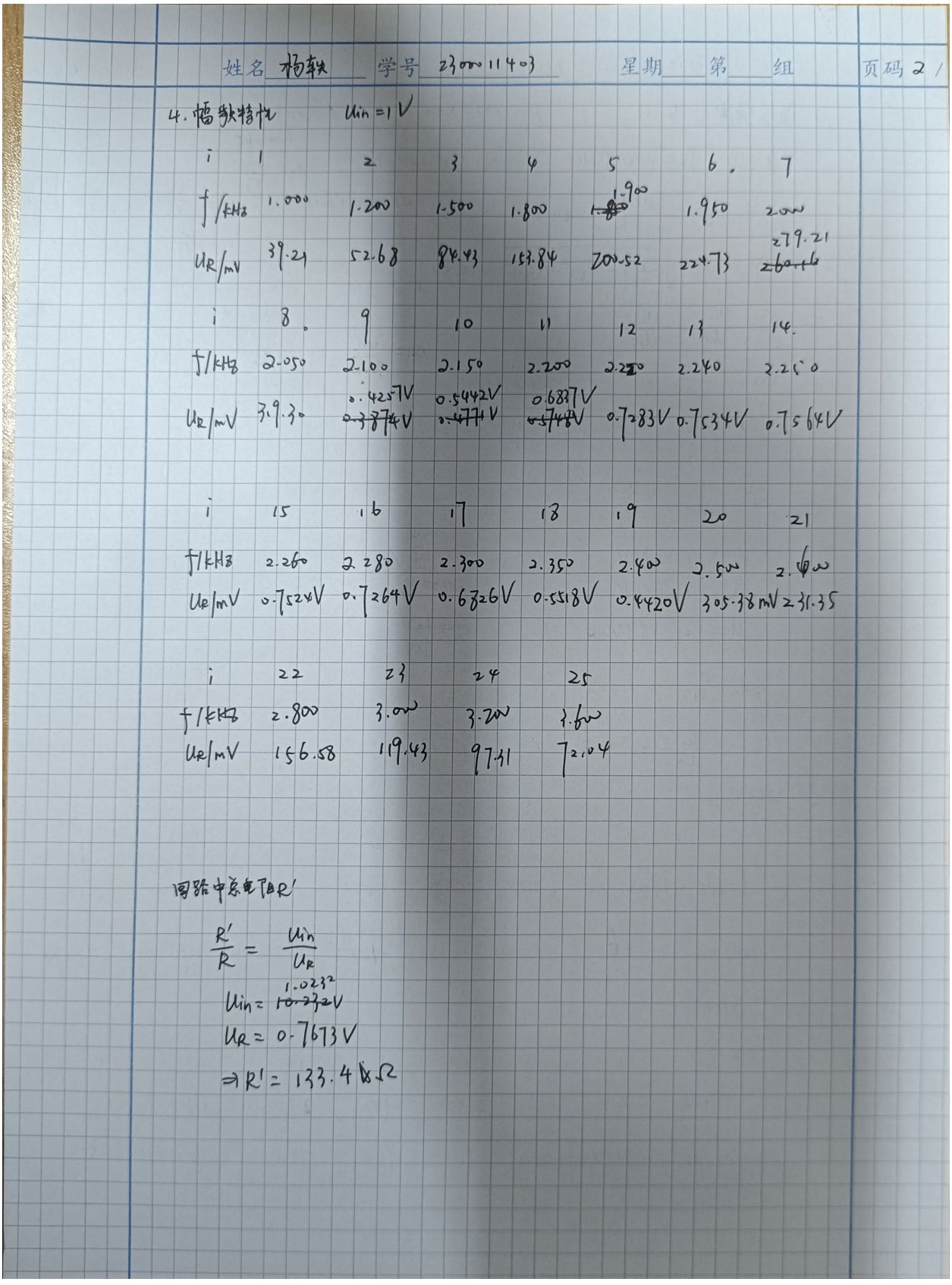
姓名 杨轶 学号 23000114003 星期 第 组 页码 2 /
4. 幅频特性 $U_{in}=1V$
i 1 2 3 4 5 6 7
f/kHz 1.000 1.200 1.500 1.800 1.900 1.950 2.000
U_R/mV 39.24 52.68 84.43 153.84 200.52 224.73 279.21
i 8 9 10 11 12 13 14
f/kHz 2.050 2.100 2.150 2.200 2.250 2.240 2.250
U_R/mV 39.30 0.4225V 0.5442V 0.6887V 0.7283V 0.7534V 0.7564V
i 15 16 17 18 19 20 21
f/kHz 2.260 2.280 2.300 2.350 2.400 2.500 2.600
U_R/mV 0.7524V 0.7264V 0.6826V 0.5518V 0.4420V 305.38mV 231.35
i 22 23 24 25
f/kHz 2.800 3.000 3.200 3.600
U_R/mV 156.58 119.43 97.31 72.04
回路中总电阻R'
$\frac{R'}{R}=\frac{U_{in}}{U_R}$
$U_{in}=1.0232V$
$U_R=0.7673V$
$\Rightarrow R'=133.4k\Omega$

图 4: 课上报告