

第五章 静磁学

§1 洛伦兹力定律

Law: 洛磁力 $\vec{F}_{\text{mag}} = Q(\vec{v} \times \vec{B})$ (\vec{B} 是屏矢量)

Add. 磁力不会做功. $\vec{F} \cdot \vec{v} = 0$ (但磁力可以作为能量转移或作用转移的途径)

电流情况下. $\vec{F}_{\text{mag}} = \int I (d\vec{l} \times \vec{B})$

推导: $\vec{I} = \lambda \vec{v} \Rightarrow \vec{F}_{\text{mag}} = \int (\vec{v} \times \vec{B}) dq = \int (\vec{v} \times \vec{B}) \lambda dl = \int (\vec{I} \times \vec{B}) dl = \int I (d\vec{l} \times \vec{B})$ $\hat{I} = d\vec{l}$

一般在根电线内, $I = I_{\text{const}} \therefore \vec{F}_{\text{mag}} = I \int d\vec{l} \times \vec{B}$

Ex. 面电流密度与受力: $\vec{K} = \frac{d\vec{I}}{dl_{\perp}}$ (垂直于电流方向单位长度上流过的电荷)

若面电荷密度为 σ , 运动速度为 \vec{v} , 则 $\vec{K} = \sigma \vec{v}$

$\vec{F}_{\text{mag}} = \int (\vec{v} \times \vec{B}) \sigma da = \int (\vec{K} \times \vec{B}) da$ (注意 \vec{B} 的变数)

(体) 电流密度与受力: $\vec{J} = \frac{d\vec{I}}{da_{\perp}}$ $\vec{J} = \rho \vec{v}$

$\vec{F}_{\text{mag}} = \int (\vec{v} \times \vec{B}) \rho d\tau = \int (\vec{J} \times \vec{B}) d\tau$

Law: 连续性方程: $\oint_s \vec{J} \cdot d\vec{a} = - \int_v \frac{\partial \rho}{\partial t} d\tau \Leftrightarrow \nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t}$

$$\oint_s \vec{J} \cdot d\vec{a} = \int_v \nabla \cdot \vec{J} d\tau = - \int_v \frac{\partial \rho}{\partial t} d\tau \Rightarrow \nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t}$$

Add them. 当 $\vec{J}|_s = \vec{0}$ 时.

$$\int_v \vec{J} d\tau = \frac{d\vec{P}}{dt}$$

证: $\vec{P} = \int_v \vec{r} \rho(\vec{r}, t) d\tau$ $\frac{d\vec{P}}{dt} = \int_v \vec{r} \frac{\partial \rho}{\partial t} d\tau = - \int_v \vec{r} (\nabla \cdot \vec{J}) d\tau$

对于 $\int_V [\vec{J} + \vec{r}(\nabla \cdot \vec{J})] d\tau$, 考虑 $(\nabla \cdot (x\vec{J}), \nabla \cdot (y\vec{J}), \nabla \cdot (z\vec{J}))$

$$\Rightarrow (J_x + x \nabla \cdot \vec{J}, J_y + y \nabla \cdot \vec{J}, J_z + z \nabla \cdot \vec{J}) = \vec{J} + (\nabla \cdot \vec{J}) \vec{r}$$

$$\Rightarrow \int_V \nabla \cdot (x\vec{J}) d\tau = \oint_S x\vec{J} \cdot d\vec{a} = 0 = \int_V \nabla \cdot (y\vec{J}) d\tau = \int_V \nabla \cdot (z\vec{J}) d\tau$$

$$\therefore \int_V (\vec{J} + \vec{r}(\nabla \cdot \vec{J})) d\tau = 0 = -\frac{d\Phi}{dt} + \int_V \vec{J} d\tau, \text{ 得证}$$

§2 毕奥-萨伐尔定律

Def. 稳恒电流: I 在导线中始终一样, $\frac{\partial \rho}{\partial t} = 0 \Leftrightarrow \nabla \cdot \vec{J} = 0$

Law: 毕奥-萨伐尔定律: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \vec{r}}{r^2} d\vec{l}' = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \vec{r}}{r^2}$ (积分沿电流路径, 按电流方向)
(- 稳恒 线电流) 且满足矢量叠加

Tip: μ_0 (真空磁导率) $= 4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2}$, \vec{B} 的单位是 $\text{T} = \text{N} \cdot (\text{A} \cdot \text{m})^{-1}$

Add: 面电流 $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{K(\vec{r}') \times \vec{r}}{r^2} da'$

体电流 $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{J(\vec{r}') \times \vec{r}}{r^2} d\tau'$

§3 \vec{B} 的散度与旋度 (稳恒电流)

Law: $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}} = \mu_0 \int \vec{J} \cdot d\vec{a} \Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$

$\boxed{\oint \vec{B} \cdot d\vec{a} = 0} \Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$

证: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \vec{r}}{r^2} d\tau'$

$$\nabla \times \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \times (\vec{J}(\vec{r}') \times \frac{\vec{r}}{r^2}) d\tau'$$

$$\nabla \times (\vec{J} \times \frac{\vec{r}}{r^2}) = (\frac{\vec{r}}{r^2} \cdot \nabla) \vec{J} - (\vec{J} \cdot \nabla) \frac{\vec{r}}{r^2} + \vec{J} (\nabla \cdot \frac{\vec{r}}{r^2}) - \frac{\vec{r}}{r^2} (\nabla \cdot \vec{J})$$

$$= \vec{J} \cdot \nabla \left(\frac{1}{r^2} \right) - (\vec{J} \cdot \nabla) \frac{1}{r^2}$$

$$\nabla \cdot \left(\frac{\vec{r}}{r^3} \right) = 4\pi \delta^3(\vec{r})$$

对 $(\vec{J} \cdot \nabla) \frac{1}{r^2}$ 的散度予分解到三个方向, 考虑 x 方向为 $(\vec{J} \cdot \nabla) \frac{x-x'}{r^3}$

$$(\vec{J} \cdot \nabla) \frac{x-x'}{r^3} = -(\vec{J} \cdot \nabla') \frac{x-x'}{r^3} = - \left[\nabla' \cdot \left(\frac{x-x'}{r^3} \vec{J} \right) + \frac{x-x'}{r^3} \nabla' \cdot \vec{J} \right] \xrightarrow{\text{为0, 忽略}} \\ \text{如利用高斯定理} = -\nabla' \cdot \left(\frac{x-x'}{r^3} \vec{J} \right)$$

$$\text{则 } \int [(\vec{J} \cdot \nabla) \frac{1}{r^2}]_x d\tau' = \int \nabla' \cdot \left(\frac{x-x'}{r^3} \vec{J} \right) d\tau' = \oint_S \frac{x-x'}{r^3} \vec{J} \cdot d\vec{a} \quad \text{取全空间, 修改为 } \vec{J} \cdot \vec{e}_x$$

$$\Rightarrow \nabla \times \vec{B} = \frac{\mu_0}{4\pi} \int \vec{J} 4\pi \delta^3(\vec{r}) d\tau' = \mu_0 \vec{J}(\vec{r}) \Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \cdot (\vec{J}(\vec{r}') \times \frac{\vec{r}}{r^2}) d\tau'$$

$$\nabla \cdot (\vec{J} \times \frac{\vec{r}}{r^2}) = \frac{1}{r^2} \cdot (\nabla \times \vec{J}) - \vec{J} \cdot (\nabla \times \frac{\vec{r}}{r^2})$$

$$\nabla \times \left(\frac{\vec{r}}{r^2} \right) = \frac{1}{r^2} (\nabla \times \vec{r}) - \vec{r} \times (\nabla \frac{1}{r^2}) = +2\hat{r} \times \hat{r} \times \frac{1}{r^3} = \vec{0}$$

$$\therefore \nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int 0 d\tau' = 0$$

并未
用到
 $\nabla \cdot \vec{J}(\vec{r})$
!!
0

§4 磁矢势

Def. 磁矢势: 根据静磁学中 $\nabla \cdot \vec{B} = 0 \Rightarrow \exists \vec{A}$, s.t. $\vec{B} = \nabla \times \vec{A}$

Tip. 然而 \vec{A} 上可加上 ∇ 梯度场 即 $\vec{A} = \vec{A}_0 + \nabla u$ $\nabla \times \vec{A} = \nabla \times \vec{A}_0 + \nabla \times (\nabla u) = \nabla \times \vec{A}_0$

$$\nabla \cdot \vec{A} = \nabla \cdot \vec{A}_0 + \nabla \cdot (\nabla u)$$

$$\text{然而此时 } \nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \nabla \times (\nabla \times \vec{A}) = \nabla \cdot (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\text{若取 } \nabla \cdot \vec{A} = 0 \Rightarrow -\nabla^2 \vec{A} = \mu_0 \vec{J}$$

此时, 若任意选取的 \vec{A}_0 , $\nabla \cdot \vec{A}_0 \neq 0$, 则在 \vec{A}_0 上加上 $\nabla \lambda$, s.t.

$$\nabla \cdot \vec{A}_0 + \nabla \cdot (\nabla \lambda) = 0 \Leftrightarrow \nabla^2 \lambda = -\nabla \cdot \vec{A}_0 \quad (\nabla^2 \phi = -\frac{\rho}{\epsilon})$$

若有 当 $r \rightarrow \infty$ 时 $\phi \rightarrow 0 \Rightarrow \phi(\vec{r}) = \frac{1}{4\pi\epsilon} \int \frac{\rho}{r} d\tau'$

同样若有 $r \rightarrow \infty$ 时 $\nabla \cdot \vec{A}_0 = 0 \Rightarrow \lambda(\vec{r}) = \frac{1}{4\pi} \int \frac{\nabla \cdot \vec{A}_0}{r} d\tau'$

但不变的是, $\begin{cases} \nabla \times \vec{A} = \vec{B} \\ \nabla \cdot \vec{A} = 0 \end{cases}$ 的解一定存在. 此时有 $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ (or $(\nabla \cdot \nabla) \vec{A} = -\mu_0 \vec{J}$)

而上式 \Leftrightarrow 3个泊松方程. 若在 $r \rightarrow \infty$ 时, $\vec{J} \rightarrow 0$ 则

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{r} dl' = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}')}{r} da'$$

Extra Solution: $\oint \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{a} = \int_S \vec{B} \cdot d\vec{a} = \Phi$

$\therefore \nabla \times \vec{A} = \vec{B} \Leftrightarrow \oint \vec{A} \cdot d\vec{l} = \Phi$. 所以用类似安培环路定理求 \vec{A}

Thm. 对第 \vec{B} , $\vec{A}(\vec{r}) = -\frac{1}{2}(\vec{r} \times \vec{B})$ 满足 $\begin{cases} \nabla \cdot \vec{A} = 0 \\ \nabla \times \vec{A} = \vec{B} \end{cases}$

Conclusion. 静磁学的边界条件.

$$\begin{aligned} \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' \\ \vec{B} &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \vec{r}}{r^3} d\tau' \\ \vec{A} &= \frac{1}{4\pi} \int \frac{\vec{B}(\vec{r}') \times \vec{r}}{r^2} d\tau' \\ \nabla \cdot \vec{A} &= 0 \quad \nabla \times \vec{A} = \vec{B} \end{aligned}$$

or $-\nabla^2 \vec{B} = \mu_0 \nabla \times \vec{J}$

$\Rightarrow \nabla^2 B_x + \mu_0 (\nabla \times \vec{J})_x = 0$ 与三个类似泊松方程 $\nabla^2 \phi = -\frac{\rho}{\epsilon}$

1° $\vec{B} = + \int \frac{\mu_0}{4\pi} \frac{\nabla \times \vec{J}}{r} d\tau'$ 且在无穷远处 $\vec{B} = 0$

2° $\nabla \times \vec{u} = \vec{J}$, $\oint \vec{u} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$ 且在有限区域 $\vec{u} = 0$

\vec{u} 为原磁场的标量流 \vec{u} 为磁矢势

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \vec{r}}{r^2} d\tau' \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' \quad \left(\begin{aligned} \nabla \times (\frac{\vec{B}}{\mu_0} - \vec{A}) &= 0 \\ \nabla \cdot (\frac{\vec{B}}{\mu_0} - \vec{A}) &= \frac{I_m}{\mu_0} \Rightarrow \vec{B} = \mu_0 \vec{A} + \frac{1}{4\pi} \int \frac{I_m \vec{r}}{r^2} d\tau' \end{aligned} \right)$$

$$\text{根据} \quad \begin{cases} \nabla \times \vec{B} = \mu_0 \vec{J} \\ \nabla \cdot \vec{B} = 0 \end{cases} \quad \begin{cases} \nabla \times \vec{A} = \vec{B} \\ \nabla \cdot \vec{A} = 0 \end{cases} \quad \vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \vec{r}}{r^2} d\tau' \Rightarrow \vec{A} = \frac{1}{4\pi} \int \frac{\vec{B}(\vec{r}') \times \vec{r}}{r^2} d\tau'$$

3' 毕安方程的方程

Thm. 边界条件

$$\vec{B} \text{ (I) 法向分量: } \oint_S \vec{B} \cdot d\vec{\sigma} = 0 \Rightarrow B_{\perp}^1 = B_{\perp}^2$$

$$\text{(II) 平行分量 } \mu_0 K dl = (B_{\perp}'' - B_{\perp}') dl \Rightarrow \mu_0 K = B_{\perp}'' - B_{\perp}' \text{ 而平行于电流方向无变化}$$

$$\text{总的 } \vec{B}_{\perp} - \vec{B}_{\perp}' = \mu_0 \vec{K} \times \vec{n} \quad (\vec{B}_{\perp} \text{ 为外面的矢量, } \vec{n} \text{ 由内指出})$$

$$\vec{A} \text{ 由于 } \oint \vec{A} \cdot d\vec{r} = \Phi \text{ 而 } \Phi \text{ 与 } S \text{ 成正比, 所以不仅 } \oint_S \vec{A} \cdot d\vec{\sigma} = 0, \text{ 又有 } \Phi \rightarrow 0, \oint \vec{A} \cdot d\vec{r} = 0$$

$$\therefore \vec{A}_{\perp} = \vec{A}_{\perp}'$$

$$\text{但根据 } \frac{\partial \phi_{\perp}}{\partial n} - \frac{\partial \phi_{\perp}'}{\partial n} = -\sigma/\epsilon_0, \text{ 三个 } A \text{ 分量均有类似关系 } \Rightarrow \frac{\partial \vec{A}_{\perp}}{\partial n} - \frac{\partial \vec{A}_{\perp}'}{\partial n} = -\mu_0 \vec{K}$$

Especially. 矢势的多极展开

Thm. 矢势的多极展开: 由于矢势和电势的相似性

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r} d\tau' = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{+\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta) \rho(\vec{r}') d\tau', \quad \theta' = \langle \vec{r}', \vec{r} \rangle$$

$$\text{则 } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \sum_{n=0}^{+\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta) \vec{J}(\vec{r}') d\tau', \quad \theta' = \langle \vec{r}', \vec{r} \rangle$$

且当 $r \gg$ 电荷分布线度时, 可用前几项拟合

$$\text{Especially. 偶极项, } \vec{A}_{\text{偶}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^2} \int r' \cos\theta' \vec{J}(\vec{r}') d\tau'$$

$$\text{当电流在一个闭合圈上时, } \vec{J}(\vec{r}') d\tau' = I d\vec{l}'$$

$$\Rightarrow \vec{A}_{\text{eq}}(\vec{r}) = \frac{\mu_0 I}{4\pi r^2} \int r' \cos\theta' d\vec{l}' = \frac{\mu_0 I}{4\pi r^2} \oint \vec{\sigma} \cdot \hat{r} d\vec{l}'$$

$$\oint (\vec{r}' \cdot \hat{r}) d\vec{l}' = - \int_S \nabla'(\vec{r}' \cdot \hat{r}) \times d\vec{a}' = - \int_S (\hat{r} \cdot \nabla') \vec{r}' \times d\vec{a} = -\hat{r} \times \int_S d\vec{a} = \vec{\sigma} \times \hat{r}$$

$$\therefore \vec{A}_{\text{eq}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{(\vec{I} \vec{a}) \times \hat{r}}{r^2}$$

Def. 磁偶极矩, $\vec{m} \equiv I \int d\vec{a} = I \vec{a}$

$$\therefore \vec{A}_{\text{eq}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Thm. 磁偶极产生的磁场. $\vec{B}_{\text{eq}}(\vec{r}) = \nabla \times \vec{A}_{\text{eq}}(\vec{r}) = \frac{\mu_0 m}{4\pi r^3} (2\cos\theta \hat{r} + \sin\theta \hat{\theta})$
 $\vec{m} = m \hat{z}$

$$\text{Especially, } \vec{B}_{\text{eq}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m}]$$

$$\text{证: } \vec{m} = (\vec{m} \cdot \hat{r}) \hat{r} + (\vec{m} \cdot \hat{\theta}) \hat{\theta}, \quad 3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m} = 2(\vec{m} \cdot \hat{r}) \hat{r} - (\vec{m} \cdot \hat{\theta}) \hat{\theta} \\ = 2m \cos\theta \hat{r} + m \sin\theta \hat{\theta}$$

$$\Rightarrow \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m}] = \frac{\mu_0 m}{4\pi r^3} [2\cos\theta \hat{r} + \sin\theta \hat{\theta}]$$

Def. 磁偶极矩: 若导线面体电流密度分别为 $\vec{I}, \vec{K}, \vec{J}$, 则给定导体磁偶极矩 \vec{m} 为

$$\vec{m} = \frac{1}{2} \int_V \vec{r}' \times \vec{I} d\vec{l} = \frac{1}{2} \int_S \vec{r}' \times \vec{K} d\vec{a} = \frac{1}{2} \int_V \vec{r}' \times \vec{J} d\vec{\tau}$$



磁势与磁流法. (注意, $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ 恒成立, 仅从 $\vec{M}(\vec{r})$ or $\vec{J}(\vec{r})$ 本构入了磁矢势

$$\begin{cases} \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{H} = \vec{J}_f \quad (\text{静磁场}) \end{cases}$$

将 Maxwell 方程仅用 \vec{H} 或 \vec{B} 表示

$$\begin{cases} \nabla \times \vec{H} = \vec{J}_f \\ \nabla \cdot \vec{H} = -\nabla \cdot \vec{M} \end{cases}$$

$$\begin{cases} \nabla \times \vec{H} = 0 \\ \nabla \cdot \vec{H} = -\nabla \cdot \vec{M} = \ell_m \rightarrow \text{磁流} \end{cases}$$

($\nabla \times \vec{H} = 0$, 保证无旋!)
 \Rightarrow 引入标势 ϕ_m , $\vec{H} = -\nabla \phi_m$
 $\nabla^2 \phi_m = -\ell_m$, $\ell_m = -\nabla \cdot \vec{M}$

$$\begin{cases} \nabla \times \vec{B} = \mu_0 (\vec{J}_f + \nabla \times \vec{M}) \\ \nabla \cdot \vec{B} = 0 \end{cases}$$

$$\begin{cases} \nabla \times \vec{B} = \mu_0 (\nabla \times \vec{M}) = \mu_0 \vec{J}_m \rightarrow \text{磁流} \\ \nabla \cdot \vec{B} = 0 \end{cases}$$

(V)

倘若在所求区域(单连通)无 \vec{J}_f , 那么有 (在此下才有磁标势与磁流法)

且磁标势仅针对 \vec{H} , 磁流法仅针对 \vec{B}

$$\text{磁荷正边界由 } (\vec{B}_1 - \vec{B}_2) \cdot \hat{n} = 0 \Rightarrow [(\vec{H}_1 + \vec{M}_1) - (\vec{H}_2 + \vec{M}_2)] \cdot \hat{n} = 0 \quad \text{or} \quad \begin{cases} \phi_1 = \phi_2 \\ \frac{\partial \phi_2}{\partial n} - \frac{\partial \phi_1}{\partial n} = (\vec{M}_2 - \vec{M}_1) \cdot \hat{n} = -\sigma_m \end{cases}$$

$$\text{在无铁磁介质时, } \vec{B} = \mu \vec{H}, \quad \nabla \times \vec{H} = 0, \quad \nabla \cdot \vec{H} = 0, \quad \vec{H} \neq 0 \Rightarrow \nabla^2 \phi_m = 0, \quad \phi_{m1} = \phi_{m2}, \quad \mu_1 \frac{\partial \phi_{m1}}{\partial n} = \mu_2 \frac{\partial \phi_{m2}}{\partial n}$$

Ex. 一个永久磁化球中. $\vec{M} = \vec{M}_0 + (\mu_r - 1) \vec{H}$ ($\vec{J}_f = 0$)

$$\vec{B} = \mu (\vec{H} + \vec{M}), \quad \nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0. \quad \text{因此仍有 } \nabla^2 \phi_m = 0$$

$$\text{在球外, } \vec{M} = 0, \quad \Rightarrow \nabla \cdot \vec{H} = 0 \quad \text{仍有 } \nabla^2 \phi_m = 0$$

$$\text{边界} \begin{cases} \phi_m|_{r=R_-} = \phi_m|_{r=R_+} \\ \frac{\partial \phi_m}{\partial n}|_{r=R_+} - \frac{\partial \phi_m}{\partial n}|_{r=R_-} = [-\vec{M}_0 + (\mu_r - 1) \vec{H}] \cdot \hat{n} \Rightarrow \mu_r \frac{\partial \phi_m}{\partial n}|_{r=R_-} - \frac{\partial \phi_m}{\partial n}|_{r=R_+} = \vec{M}_0 \cdot \hat{n} \end{cases}$$

$$\text{角点} \begin{cases} \phi_m|_{r=0} \text{ 有限} \\ \phi_m|_{r \rightarrow \infty} = 0 \end{cases}$$